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Minimize the Prediction Error in External Consensus Problem using Gain Error Ratio Formula

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Abstract: This paper discusses the external consensus problem for non-identical networked multi-agent systems (NMAS) with network data loss, considering uniform consecutive data losses (CDL) induced by long periods of transmission failure. A gain error ratio (GER) formula is proposed to determine the appropriate value of coupling gain between agents in order to minimize the computed prediction error caused by the prediction process. Consequently, the consensus performance with prediction control strategy can be improved. The effectiveness of the proposed formula is demonstrated through simulation.

Keywords: Consecutive data losses; consensus; coupling gain; multi-agent system; prediction.

1. INTRODUCTION

In recent years, networked multi-agent systems (NMAS) has attracts significant interest within the control research community. This can be seen by numerous published findings which has been markedly active and it has been increasing over the last few years. NMAS has given great advantages over the conventional wired control structure with the utilization of the network communication, for example, higher flexibility and modularity. This circumstance has started the enthusiasm of researchers to utilize the NMAS structure in various multi-disciplinary applications. However, the utilization of network communication in the NMAS has certainly introduces inherent constraints such as network delay and data losses which caused by limited bandwidth and overhead in the network communication. Many studies have been focused only on the presence of network delay while occurrence of data losses generally is not completely explored. Along these lines, in this paper, only data loss will be considered. Though that cooperation among NMAS agents is performed through a shared network communication, it may be tough to guarantee that all transmitted data will be successfully received by the neighboring agent(s).

In practical applications, because the network communication condition is highly dependent upon the rate of usage, it will be not ceaselessly stable; the network can sometimes be coincidentally disabled for a few moments. It is thus common for transmitted data to be lost sometimes. When data is delayed for a long period of time (surpassing the maximum allowable network delay), it is recognized as circumstance of data dropout. The impact of data losses and the maximum allowable consecutive data losses need to be verified to guarantee stable NMAS performance. The significance impact could be seen when data loss is happened for a long period which might result in substantial consecutive data losses.

Consensus is one sort of the cooperative control over NMAS which involves various agents communicated with one another which will meet towards a common value upon request. The event of data losses over NMAS will significantly degrade the consensus performance. Many efforts have been devoted to compensate the data loss impacts for NMAS and to enhance the consensus convergence performance which can be found in [1-5]. However, very little effort has been made to investigate about coupling gain and its positive impact on consensus convergence performance if the optimum value is applied.

In this paper, the model predictive control with Gain Error Ratio (GER) formula is presented. The comparison between coupling gain based-GER and without GER in solving the data loss problem is given. The aim of this work is to show the advantages of applying the optimum coupling gains in solving the external consensus problem with data loss compensation. Only basic mathematical calculation is required. To explore the capability of GER formula, uniform consecutive data loss is considered.

2. PROBLEM FORMULATION

2.1 Preliminaries Theory

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ be an undirected graph of order n with the set of nodes or agents $\mathcal{V} = \{v_1, v_2, .., v_n\}$ and edges, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. An edge from i to j is denoted by $e = (v_i, v_j)$ indicate that agent j can receive information from agent i and vice versa. In undirected graph, an edge from i to j and j to i has no exact direction which has positive unweighted adjacency matrix, $a_{ij} = a_{ji} = 1$ for all i,j. No self-loop is allowed, hence $a_{ii} = a_{jj} = 0$. The set of neighbour agent i is denoted by $N_i = \{j \in \mathcal{V}: (j,i) \in \mathcal{E}\}$. The Laplacian matrix L with respect to undirected graph $\mathcal{G}$ can be simply obtained as

$$L = [l_{ij}]_{n\times n}$$

where

$$l_{ij} = \begin{cases} |N_i|, & i = j \\ -1, & N_i \\ 0, & otherwise \end{cases}$$

However, in this work, the Laplacian matrix L has non-zero elements because every agent is interconnected to one another. Obviously, row-sums of L will be zero. Therefore, zero eigenvalue of L, $\lambda_1 = 0$ will be the smallest eigenvalue if and only if $\mathcal{G}$ has a spanning tree and $\mathcal{G}$ is strongly connected. The second smallest eigenvalue of L, $\lambda_2 > 0$ if and only if the $\mathcal{G}$ is connected.

2.2 Uniform CDL

A uniform CDL is the data losses that occurring at uniform intervals. This type of data losses condition can be represented by data loss simulation model which consists of basic blocks in MATLAB SIMULINK as in Figure 1.

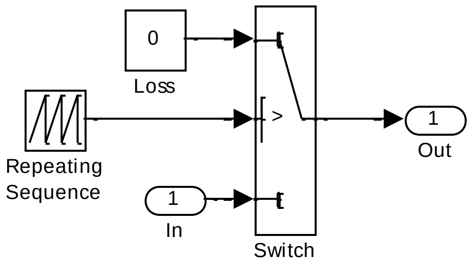


Figure 1. Data loss simulation model for uniform CDL

The output of data loss simulation model can be expressed as:

$$Uniform\ CDL;\ Out = \begin{cases} 0\ if\ r_s > threshold\ value \\ In\ if\ r_s \leq threshold\ value \end{cases}$$

where In is the input to the data loss simulation model which is the prediction sequence of agent j and *threshold value* is the pre-set value. The switch acts as a network transmission line to replicate the scenarios of the network operating with and without data loss. The switch propagates one of two inputs (either loss (0) or no-loss (*In*)) triggered by the value of the control input r_s . The pre-set value is the value of the control input r_s at which the switch flips to its other input. The control input r_s is represented by a set of repeating sequence signal to generate the situation of uniform CDL.

2.3 External Consensus Law

There are three common methods used to solve the data loss problem such as zero input, past value and predicted value. In this paper, the external consensus law based on predicted value is proposed which can be described as follows:

$$u_i(k) = \begin{cases} G_1(z^{-1})\left(K_r\big(R(k) - y_1(k)\big) - \sum_{j=2}^{n} K_{1j}\left(y_1(k) - (1-\alpha_{j1})y_j(k|k-p) - \alpha_{j1}y_j(k)\right)\right) & for\ i = 1 \\ -G_i(z^{-1})\left(\sum_{j=1}^{n} K_{ij}\left(y_i(k) - (1-\alpha_{ji})y_j(k|k-p) - \alpha_{ji}y_j(k)\right)\right) & for\ i = 2,3,\dots,n \end{cases} \quad (1)$$

where K_r and K_{ij} are the best calculated constant coupling gains using the proposed GER formula which will be explained in section 2.5. The external reference input is denoted by $R(k)$, $u_i(k)$ is the control input, $y_i(k)$ and $y_j(k)$ are the measured outputs of agents i and j respectively and n represents the number of NMAS agents. The number of CDL is denoted by p.

During the loss period, the prediction output $y_j(k|k-p)$ that has been stored at agent i will be applied. The occurrence of data loss is assumed to be uniform among agents where all the agents are subject to the same data loss. The proposed consensus protocol in (1) is said to solve the external consensus problem if and only if $\lim_{k\to\infty}\|R(k) - y_1(k)\| = 0$ and $\lim_{k\to\infty}\|y_i(k) - y_j(k)\| = 0$ if there exist coupling gains for $i = 1, 2, \dots, n$.

2.4 Predictive Control

Using the algorithm in [6], the prediction sequence is generated up to the possible maximum CDL p_{max}. For $i = 1, 2, \ldots, n$, the prediction sequence is computed by agent *i* before the data is transmitted to other agent(s). The other agent(s) that receive the prediction sequence will store the sequence data and use it whenever a failure in transmission occurs.

The predictions sequence of agent *i* from time $k - p + 1$ to k for $l_{cs} = 1,2, \ldots, p_{max}$ can be summarized in terms of general equation in (2) and (3). From both equations, it can be seen that both the prediction sequence signal and the current signal available at time $k - p$ are required for the computation to be successful.

$$\begin{aligned} y_i(k-p+l_{cs}|k-p) \\ &= -\sum_{f=1}^{\min\{n_{ai},l_{cs}-1\}} a_{if} y_i(k-f-p+l_{cs}|k-p) - \sum_{f=l_{cs}}^{n_{ai}} a_{if} y_i(k-f-p+l_{cs}) \\ &+ b_{i0}\hat{u}_i(k-p|k-p) \\ &+ \sum_{f=0}^{\min\{m_{bi},l_{cs}-1\}} b_{if}\hat{u}_i(k-f-1-p+l_{cs}|k-p) + \sum_{f=l_{cs}-1}^{m_{bi}} b_{if} u_i(k-f-1-p+l_{cs}) \end{aligned} \quad (2)$$

$$\hat{u}_i(k-p+l_{cs}|k-p) = \begin{cases} -\sum_{f=1}^{\min\{n_{c1},l_{cs}-1\}} c_{1f}\hat{u}_1(k-f-p+l_{cs}|k-p) - \sum_{f=l_{cs}}^{n_{c1}} c_{1f}\, u_1(k-f-p+l_{cs}) \\ + K_r D_1(z^{-1}) R(k-p+l_{cs}) - \sum_{f=0}^{\min\{n_{d1},l_{cs}-1\}} d_{1f} K_r y_1(k-f-p+l_{cs}|k-p) \\ - \sum_{f=0}^{\min\{n_{d1},l_{cs}-1\}} d_{1f}\left(K_{1j}\sum_{\substack{j=2\\ j\neq i}}^{n}\left(y_1(k-f-p+l_{cs}|k-p) - \bar{y}_j(k-f-p+l_{cs}|k-p)\right)\right) \\ - \sum_{f=l_{cs}}^{n_{d1}} d_{1f} K_r y_1(k-f-p+l_{cs}) \\ - \sum_{f=l_{cs}}^{n_{d1}} d_{1f}\left(K_{1j}\sum_{\substack{j=2\\ j\neq i}}^{n}\left(y_1(k-f-p+l_{cs}) - \bar{y}_j(k-f-p+l_{cs})\right)\right) & for\ i = 1 \\ - \sum_{f=1}^{\min\{n_{ci},l_{cs}-1\}} c_{if}\hat{u}_i(k-f-p+l_{cs}|k-p) - \sum_{f=l_{cs}}^{n_{ci}} c_{if}\, u_i(k-f-p+l_{cs}) \\ - \sum_{f=0}^{\min\{n_{di},l_{cs}-1\}} d_{if}\left(K_{ij}\sum_{\substack{j=1\\ j\neq i}}^{n}\left(y_i(k-f-p+l_{cs}|k-p) - \bar{y}_j(k-f-p+l_{cs}|k-p)\right)\right) \\ - \sum_{f=l_{cs}}^{n_{di}} d_{if}\left(K_{ij}\sum_{\substack{j=1\\ j\neq i}}^{n}\left(y_i(k-f-p+l_{cs}) - \bar{y}_j(k-f-p+l_{cs})\right)\right) & for\ i = 2,3,\ldots,n \end{cases} \quad (3)$$

2.5 Gain Error Ratio (GER) Formula

The coupling gains K_r and K_{ij} can be calculated based on below equations:

$$K_r = \frac{\sum_{k=1st\ sample}^{end\ sample} |r(k) - y_1(k)|_{1\ cycle}}{\left(\sum_{k=1st\ sample}^{end\ sample} |r(k) - y_1(k)|_{1\ cycle} + \sum_{j=2}^{n}\sum_{k=1st\ sample}^{end\ sample} \left|y_1(k) - y_j(k)\right|_{no\ loss}\right)}$$

$$K_{ij}=\begin{cases}\dfrac{\sum_{k=1st\,sample}^{end\,sample}\left|y_i(k)-y_j(k)\right|_{no\,loss}}{\left(\sum_{k=1st\,sample}^{end\,sample}\left|r(k)-y_i(k)\right|_{1\,cycle}+\sum_{j=2}^{n}\sum_{k=1st\,sample}^{end\,sample}\left|y_i(k)-y_j(k)\right|_{no\,loss}\right)} & for\ i=1 \\ & \quad j\in N_1 \\ \dfrac{\sum_{k=1st\,sample}^{end\,sample}\left|y_i(k)-y_j(k)\right|_{no\,loss}}{\left(\sum_{j=1}^{n}\sum_{k=1st\,sample}^{end\,sample}\left|y_i(k)-y_j(k)\right|_{no\,loss}\right)} & for\ i=2,3,\dots,n \\ & \quad j\in N_i\end{cases} \tag{4}$$

where k refers to the k-th sample. These error values can be obtained using the values of $r(k)$, $y_i(k)$, and $y_j(k)$ for $i=1,2,\dots,n$ and $j\in i\cup N_i$ from the NMAS consensus result with prediction without coupling gains for uniform CDL through simulations. The result shows a repetitive pattern for every cycle (loss + no-loss) during its steady-state condition as shown an example of 16 CDL in Figure 2. Thus, to simplify this method, any one cycle during the steady-state period is chosen to calculate K_r and K_{ij}. For examples, referring to Figure 2, samples 100 to 119 are taken as one cycle during the steady-state period.

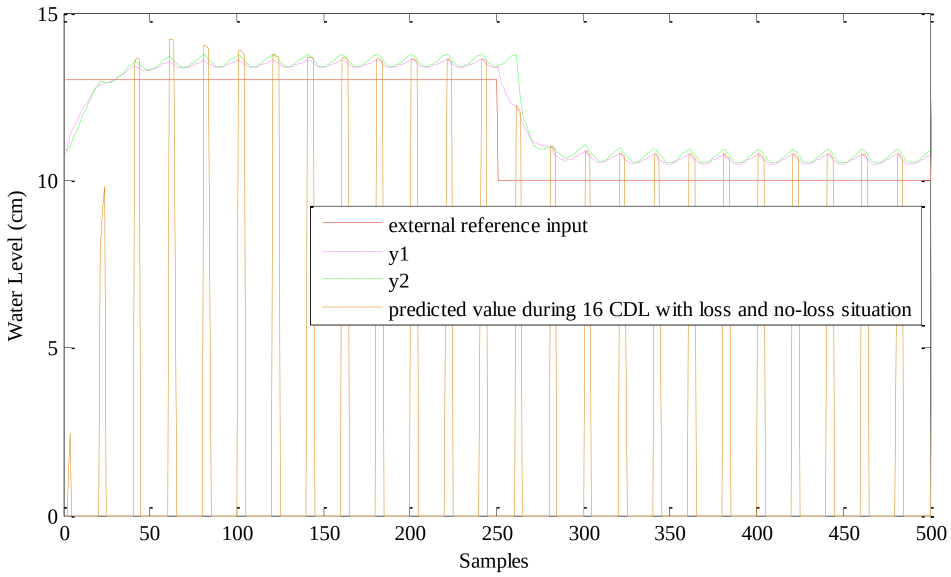


Figure 2. Consensus performance with prediction only for 16 CDL

Within these 20 samples (1 cycle), there are loss and no-loss situation at different samples. For 16 CDL, a no-loss situation occurs at samples 100 to 103 and a loss situation occurs at samples 104 to 119. Thus, the values of $r(k)$, $y_i(k)$, and $y_j(k)$ at the specified sample are used to calculate the error ratio as follows:

$$K_r=\frac{\sum_{k=101}^{120}\left|r(k)-y_i(k)\right|}{\left(\sum_{k=101}^{120}\left|r(k)-y_i(k)\right|+\sum_{j=2}^{n}\sum_{k=101}^{105}\left|y_i(k)-y_j(k)\right|\right)}$$

$$K_{ij}=\begin{cases}\dfrac{\sum_{k=101}^{105}\left|y_i(k)-y_j(k)\right|}{\left(\sum_{k=101}^{120}\left|r(k)-y_i(k)\right|+\sum_{j=2}^{n}\sum_{k=101}^{105}\left|y_i(k)-y_j(k)\right|\right)} & for\ i=1 \\ & j\in N_1 \\ \dfrac{\sum_{k=101}^{105}\left|y_i(k)-y_j(k)\right|}{\left(\sum_{j=1}^{n}\sum_{k=101}^{105}\left|y_i(k)-y_j(k)\right|\right)} & for\ i=2,3,\dots,n \\ & \quad j\in N_i\end{cases} \tag{5}$$

During losses, only the stored (prediction) value is used and thus the actual error value is not available. However, since the connection between the external reference input $r(k)$ and Agent 1 is not subject to loss, the error for the whole cycle is considered. For random CDL, the coupling gains calculated using (4) can be applied as long as the maximum value of CDL is known.

3. SIMULATION RESULTS

3.1 Predictive controller with GER formula for uniform CDL

In this section, a numerical example is given to illustrate the effectiveness of the proposed GER formula. Consider NMAS in a fixed topology with n =3 indexed by 1, 2 and 3 respectively. The dynamics of agent i (i = 1, 2, 3) are described by the system model in (6), where

$$P_1(z^{-1})=\frac{0.06354z^{-1}+0.00497z^{-2}}{1-0.9692z^{-1}} \tag{6}$$

$$P_2(z^{-1}) = \frac{0.08345z^{-1} + 0.0222z^{-2}}{1 - 0.7184z^{-1} + 0.01505z^{-2}}$$

$$P_2(z^{-1}) = P_3(z^{-1})$$

The transfer functions of virtual local controllers for all agents are obtained by employing the Proportional-Integral (PI) controller. The transfer functions are

$$G_{c1}(z^{-1}) = \frac{1.35 - 1.31z^{-1}}{1 - z^{-1}} \quad (7)$$

$$G_{c2}(z^{-1}) = \frac{1 - 0.6z^{-1}}{1 - z^{-1}}$$

$$G_{c2}(z^{-1}) = G_{c3}(z^{-1})$$

The consensus performance for NMAS with 3 agents is illustrated in Figure 3 with corresponding control gains are tabulated in Table 1.

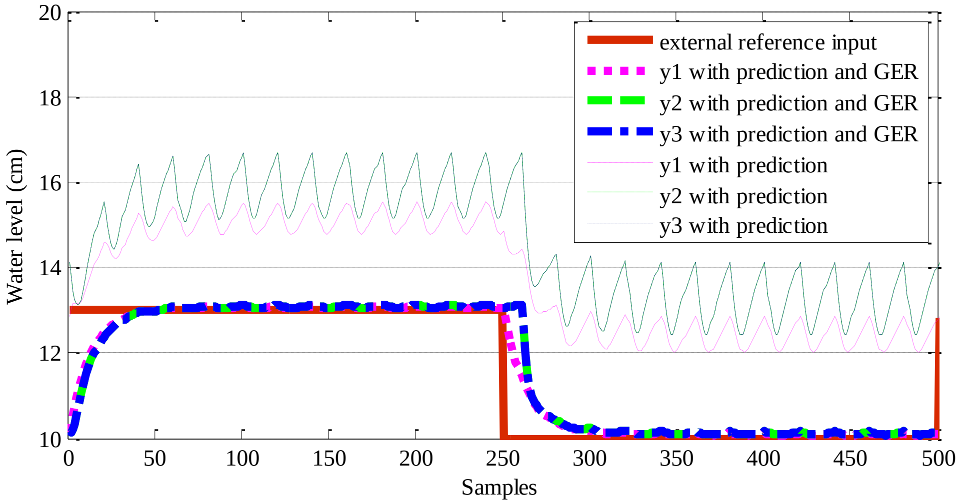


Figure 3. External consensus for NMAS with 3 agents at 16 CDL using NPCA-GER

Table 1. Calculated control gains K_r and K_{ij} using GER formula for 3 agents

Control gains	**16 CDL**
K_r	0.87
K_{12}	0.065
K_{13}	0.065
K_{21}	1
K_{23}	0
K_{31}	1
K_{32}	0

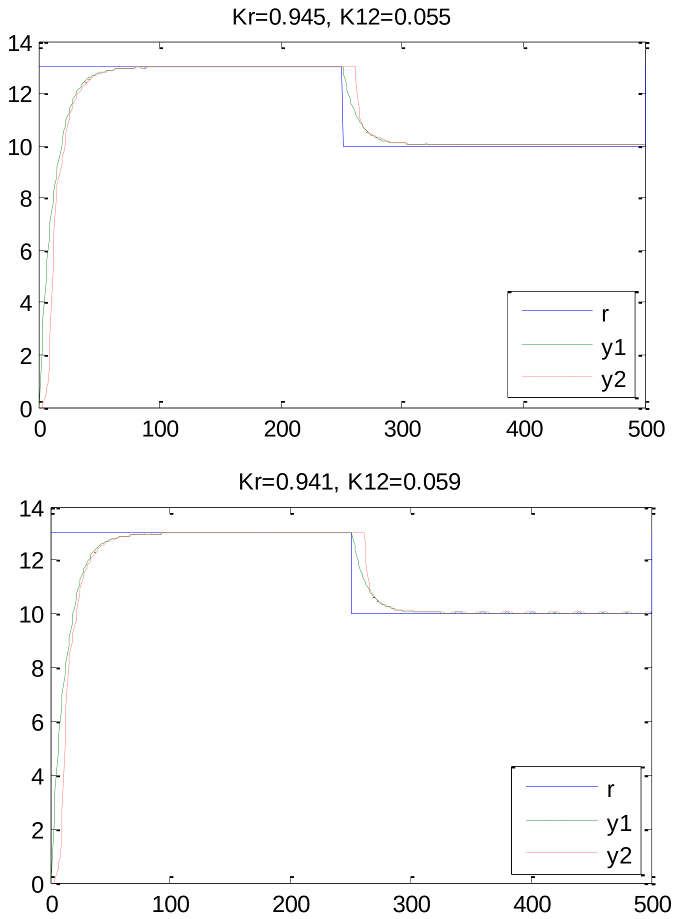
From Figure 3, it can be concluded that there is a huge improvement in the consensus performance with application of NPCA-GER. The prediction error has been significantly minimized for every agent. Since the model of Agent 2 and 3 is similar, output difference between these two agents is zero. Consequently, the calculated control gains K_{23} and K_{32} are also zero.

3.2 Comparison of calculated coupling gains at different cycle

Any steady-state cycle of the NMAS with prediction can be used to calculate the coupling gain. Even though the value of the gain will be slightly different for each cycle (as shown in Table 2), the consensus performance is almost identical as shown in Figure 4. Table 2 shows an example of calculated control gains for 16 uniform CDL at different steady-state cycle.

Table 2. Control gains at different cycle for 16 uniform CDL

Cycle	Samples (101-120)	Samples (201-220)	Samples (321-340)
K_r	0.945	0.941	0.95
K_{12}	0.055	0.059	0.05
K_{21}	1	1	1



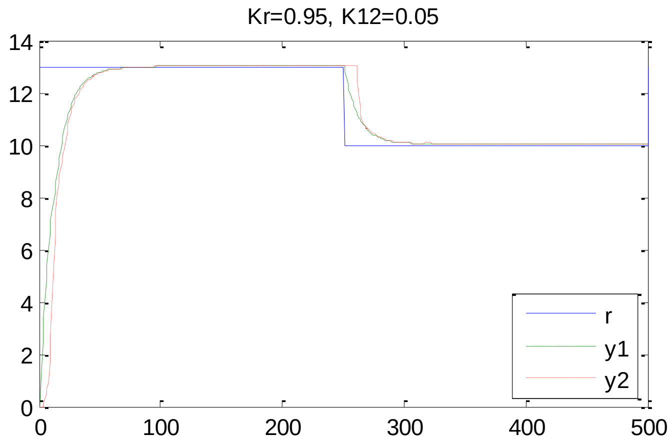


Figure 4. Consensus performance with 3 different set of gains for 16 uniform CDL (y-axis is water level (cm) and x-axis is number of samples)

Suitable cycle for GER formula is not fixed and depending on the user preference and design. In this paper, in all uniform CDL cases, cycle is fixed to 20 samples. If 9 uniform CDL in considered, this means 9 samples lost and 11 samples not lost in the cycle. The same condition is considered for 3, 13, 15 and 16 with uniform CDL.

4. CONCLUSION

This paper investigates the advantageous of having suitable coupling gain in solving the external consensus problem in the occurrence of large consecutive data loss. With the introduction of the GER, the external consensus performance of the non-identical NMAS consensus has increased by solving the imprecise long prediction computation process.

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